Smooth Kronecker

SYSTOPIA

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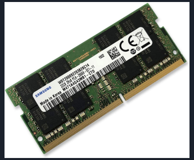
Graph Systems have taken over the world

- Graph Data is everywhere
- Exponential growth in research of distributed graph processing systems since Pregel (2010)
- Graph Processing Systems must scale to very large graphs

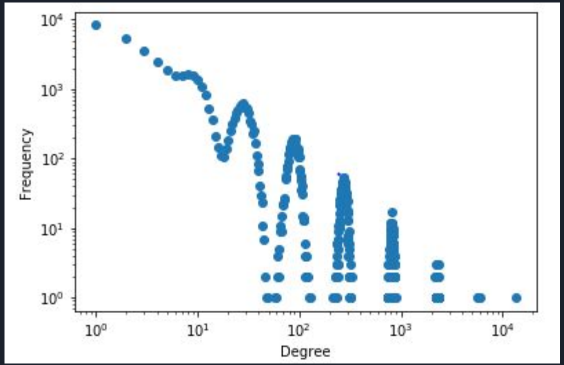
Widely used graphs are small

Dataset	# Vertices	# Edges	Size	Device whose memory dataset will fit in
cit-Patents	~3.7M	~16.5M	289MB	iPhone 4 
soc-LiveJournal	~4.8M	~68.9M	1.1GB	iPhone 7 
Twitter-2010	~41M	~1.4B	26GB	Our advisor's laptop 

Kronecker Generators save the day

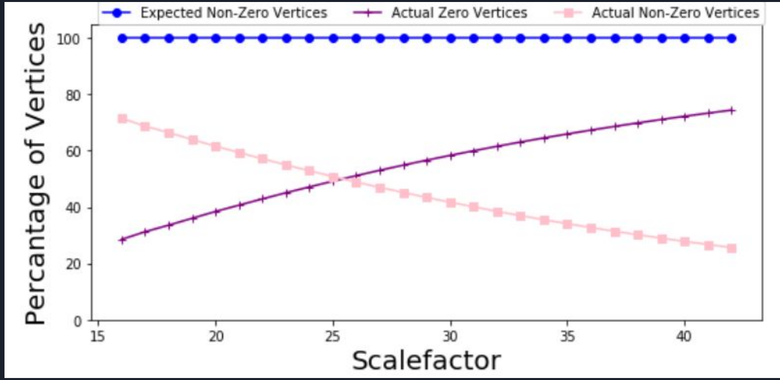
- R-Mat and Kronecker Graph generators have been popularly used for generating large graphs.
- They are easy to use and highly parallelizable which makes them perfect as the data source for scalability experiments.
- Edges in the graph are generated according to a 2 x 2 seed distribution where each element in the distribution approximately corresponds to the fraction of edges in a particular quadrant of the adjacency matrix of the graph.

Do they really save the day?

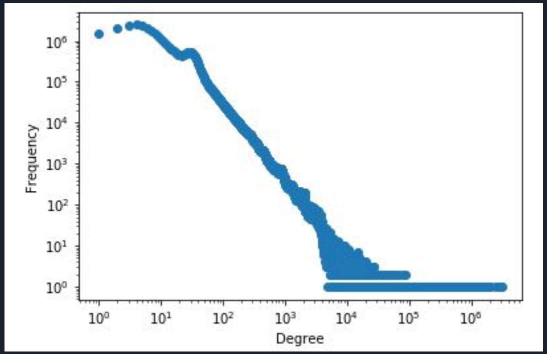


Kronecker Graph generators suffer from 2 major problems that make them inappropriate for scalability benchmarking of graph processing systems

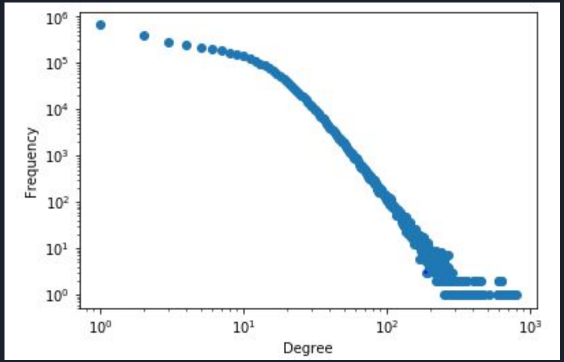
- The degree distributions of Kronecker graphs are combed, unlike any real graph
- As scale increases, so does the fraction of 0-degree vertices.
- **Implication**: a scale of 30 produces a graph with only 400M non-0 degree vertices, not the expected 1 Billion



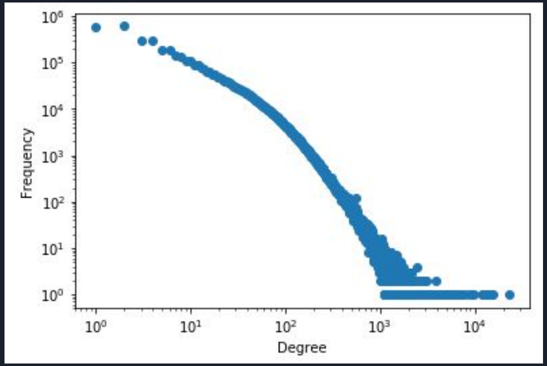
Which one of these is not like the others?



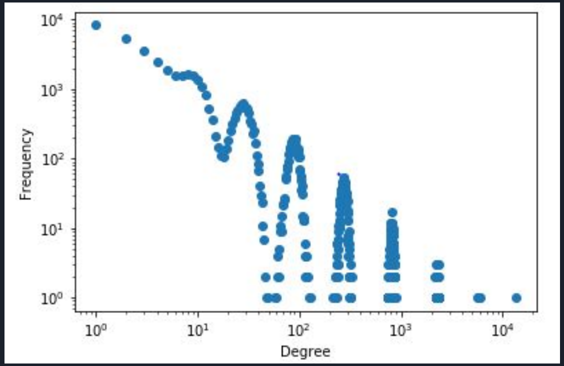
Twitter-2010



Cit-Patents



Soc-Livejournal



Kronecker (Snap)

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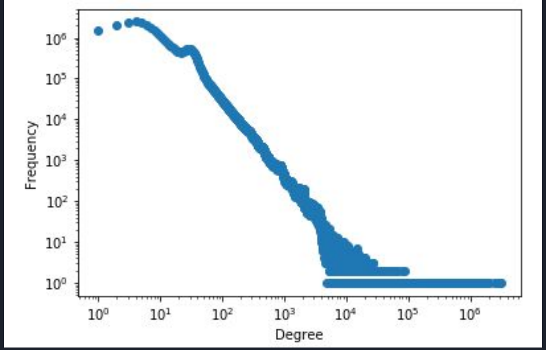

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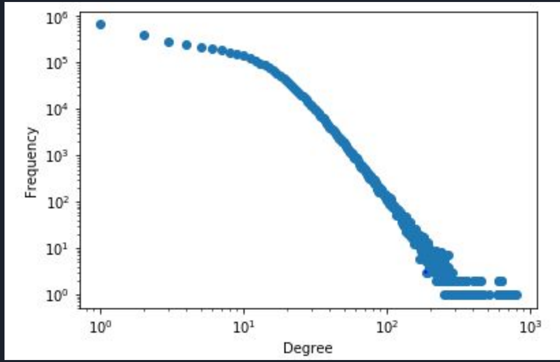

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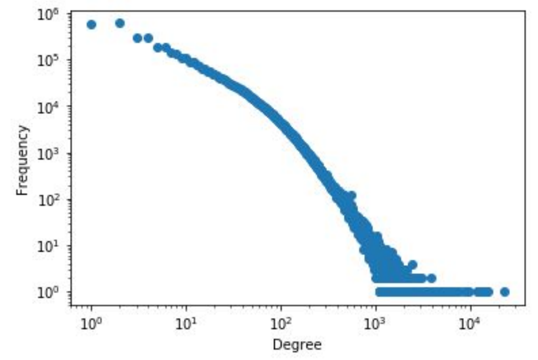

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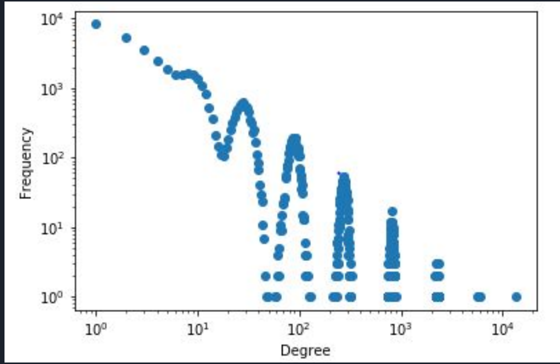
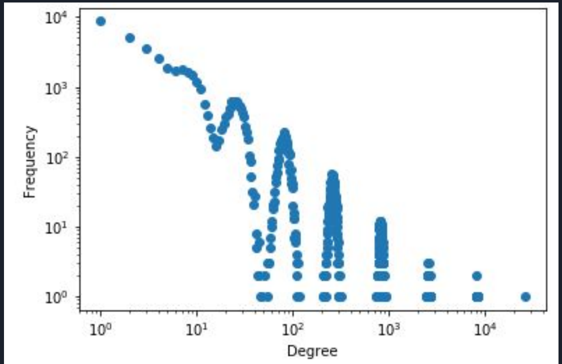
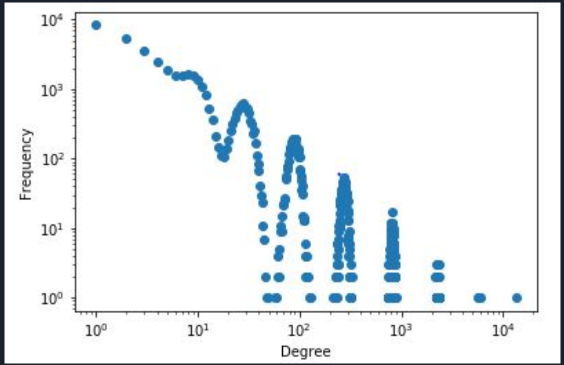


Kronecker (Snap)

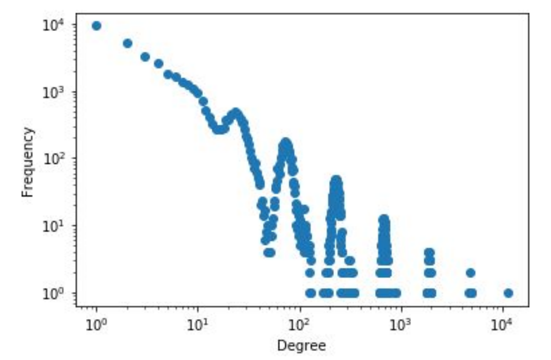
Degree distribution combing is pervasive



Graph500



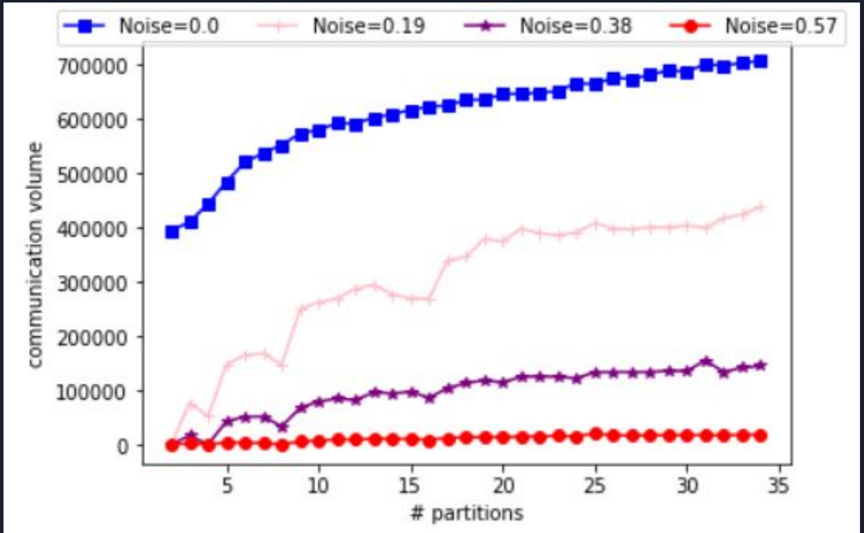
Snap-Kron



TrillionG

Adding Noise

- Seshadhri et al. proposed smoothing by blurring kronecker iterations with uniform random noise.
- It is not completely obvious how much noise to add
- Adding too much noise can be catastrophic as it can drastically affect the partitionability of the graph.



Smooth Kronecker Generator

Problems with using the Kronecker-generated graphs for benchmarking

- The Kronecker model follows a reasonable degree distribution; however it is undersampled as each vertex's degree is determined by the non-unique sum of its binary representation.
- For Noisy Kronecker, the vertex degrees vary normally around the undersampled model, instead of sampling the correct underlying model.

Solution:

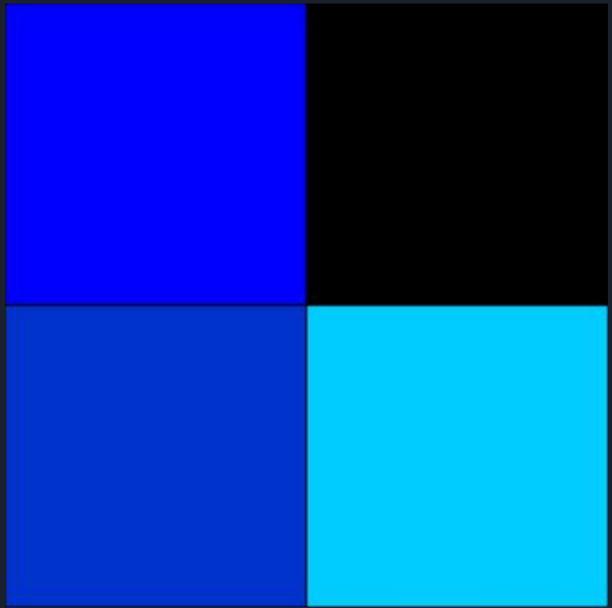
- Resample a different small distribution from the correct underlying model and mix it with the original 2x2 seed to smoothen the undersampling.

The Kronecker Product

- Mathematically, it is a product between two matrices where each element in a matrix is multiplied with the whole other matrix.

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & \cdots & a_{1n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m1}\mathbf{B} & \cdots & a_{mn}\mathbf{B} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \otimes \begin{bmatrix} 0 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 1\begin{bmatrix} 0 & 5 \\ 6 & 7 \end{bmatrix} & 2\begin{bmatrix} 0 & 5 \\ 6 & 7 \end{bmatrix} \\ 3\begin{bmatrix} 0 & 5 \\ 6 & 7 \end{bmatrix} & 4\begin{bmatrix} 0 & 5 \\ 6 & 7 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1\times 0 & 1\times 5 & 2\times 0 & 2\times 5 \\ 1\times 6 & 1\times 7 & 2\times 6 & 2\times 7 \\ 3\times 0 & 3\times 5 & 4\times 0 & 4\times 5 \\ 3\times 6 & 3\times 7 & 4\times 6 & 4\times 7 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0 & 10 \\ 6 & 7 & 12 & 14 \\ 0 & 15 & 0 & 20 \\ 18 & 21 & 24 & 28 \end{bmatrix}.$$



Kronecker multiplication in 1-Dimension

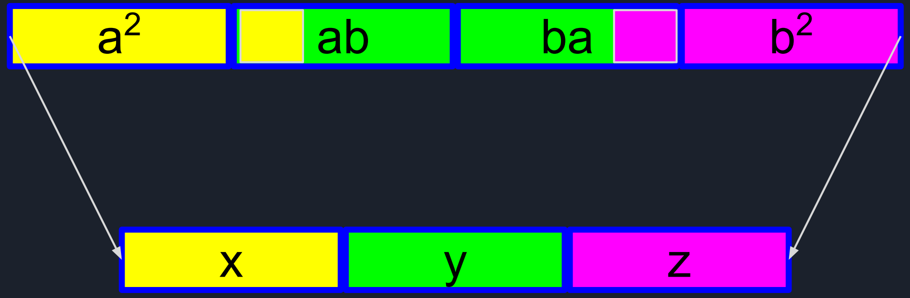
a	b

a^2	ab	ba	b^2

a^3	a^2b	a^2b	ab^2	ba^2	b^2a	b^2a	b^3

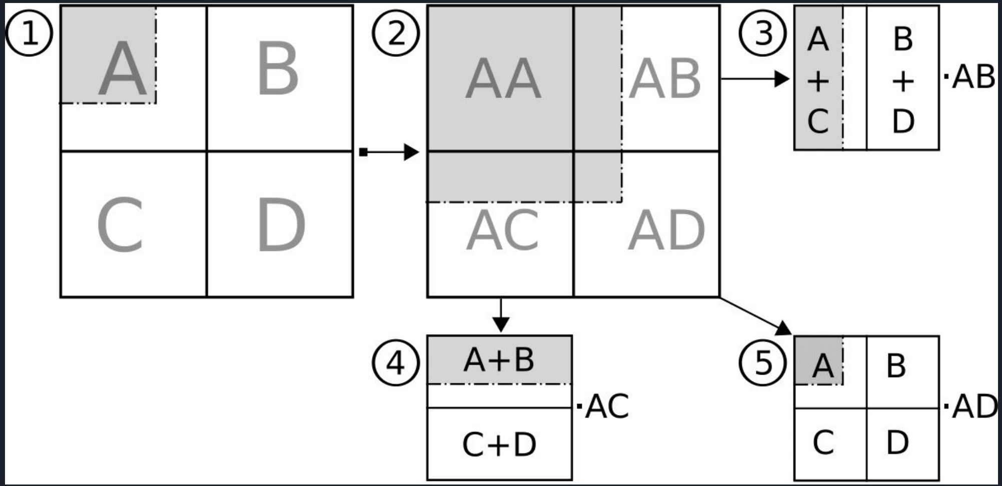
Resampling in 1-Dimension

- To resample the initial 1x2 seed [a,b] as a 1x3 seed [x,y,z], we sample the leftmost third (x), the middle third (y), and the rightmost third (z) as the Kronecker product reaches infinity.
- For the leftmost third, i.e. x, the value is just the geometric series with initial term **aa** and ratio **ab.**
- For the rightmost third, i.e. z, the value is just the geometric series with initial term **bb** and ratio **ab**
- y is simply calculated as 1-x-z.



2x2 Seed Resampling

- We now need to resample a 2x2 seed as a 3x3 seed.
- We follow the same idea from the 1D sampling where we sample each element of the 3x3 seed as the Kronecker product of the 2x2 seed goes to infinity.
- For eg: The first element is sampled as the leftmost corner goes to infinity.



Smooth Kronecker Algorithm

1. Given a 2x2 seed, number of edges, scalefactor2 and scalefactor3, first resample a 3x3 seed.

- Scalefactor2 and scalefactor3 collectively decide the number of vertices in the matrix.
- The total number of vertices is given by $2^{\text{scalefactor2}} * 3^{\text{scalefactor3}}$.
- Generate an edge using scalefactor2 + scalefactor3 Kronecker iterations.

a	b
c	d

2 x 2 Seed

r	s	t
u	v	w
x	y	z

3 x 3 Seed

Smooth Kronecker Algorithm

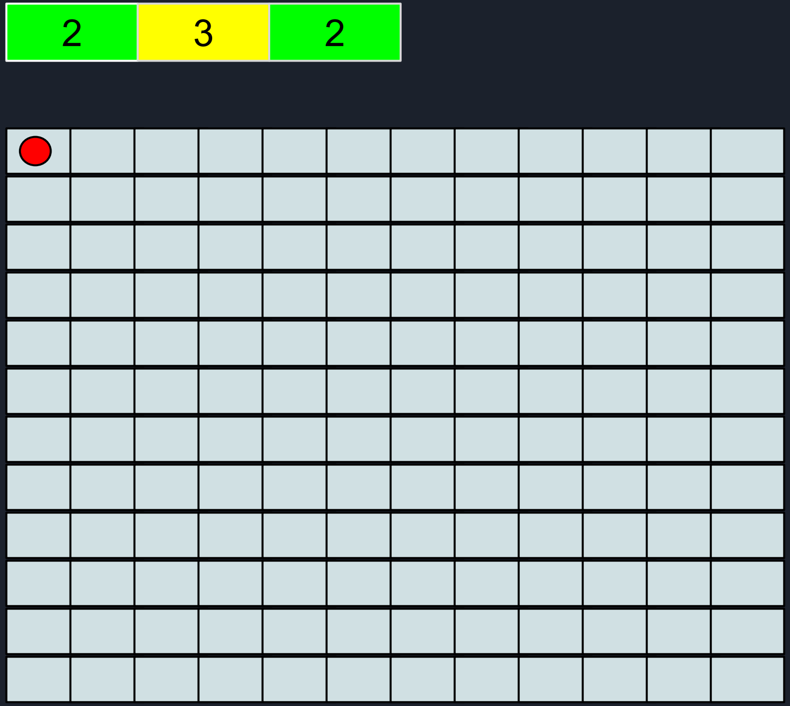
1. Given a 2x2 seed, number of edges, scalefactor2 and scalefactor3, first resample a 3x3 seed.

2. For each edge: randomly determine which seed will be used for sampling at every Kronecker iteration.

- E.g., For scalefactor2=4 and scalefactor3=2, one possible order is shown on the right.

2	Iteration #1
2	Iteration #2
2	Iteration #3
3	Iteration #4
2	Iteration #5
3	Iteration #6

Smooth Kronecker Algorithm



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3. Recursively sample from the seed according to the operation order to obtain the source and destination of the edge.

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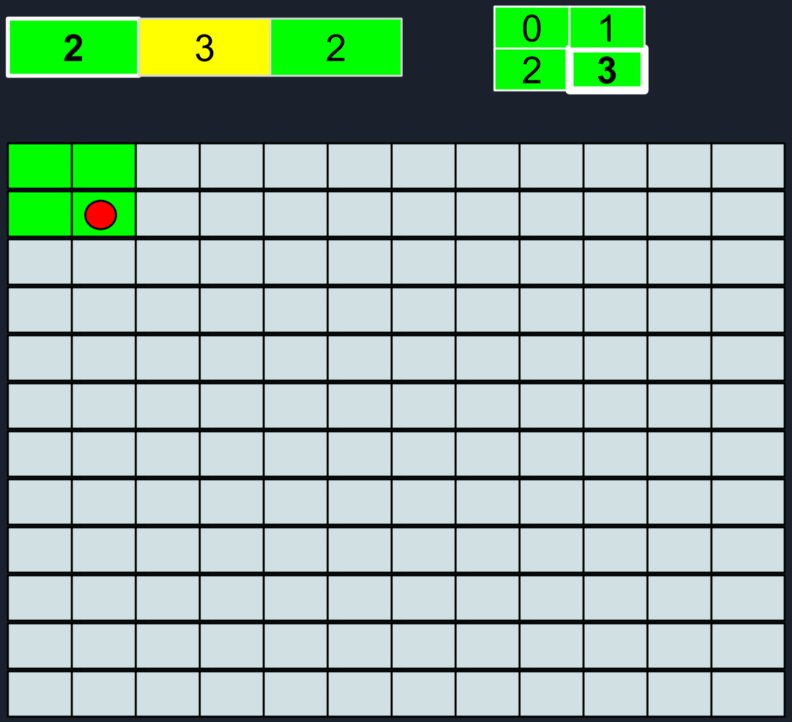

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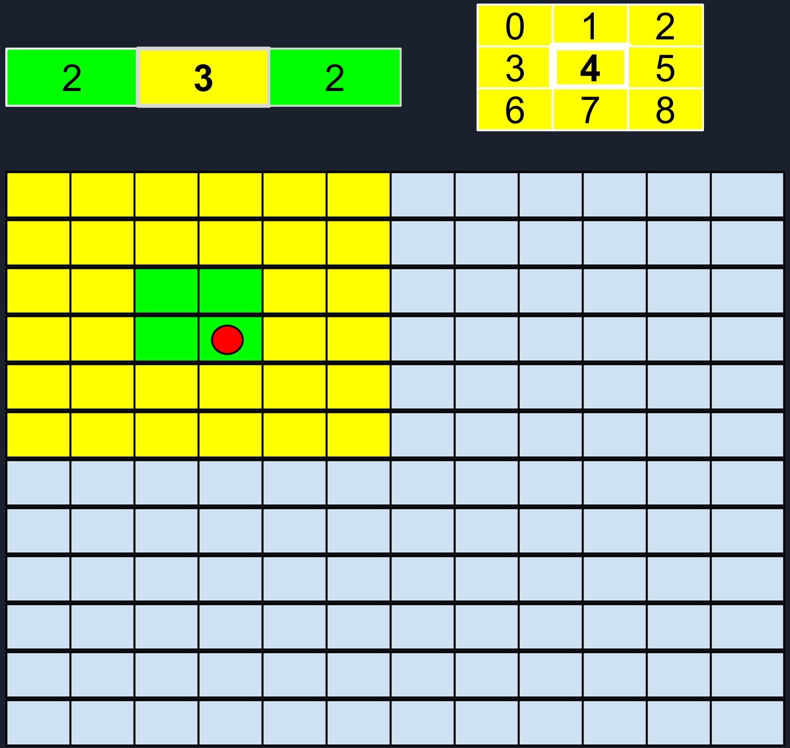

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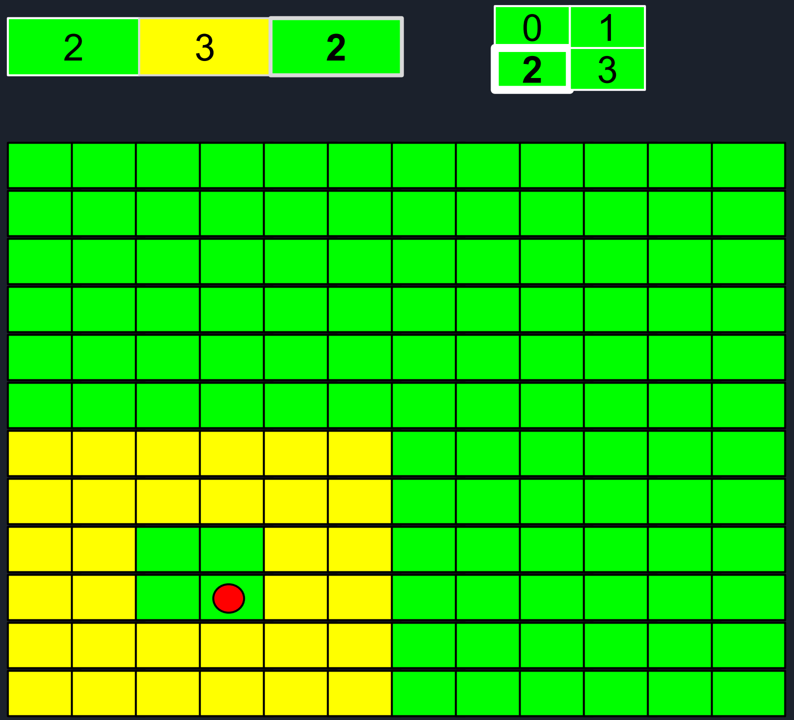
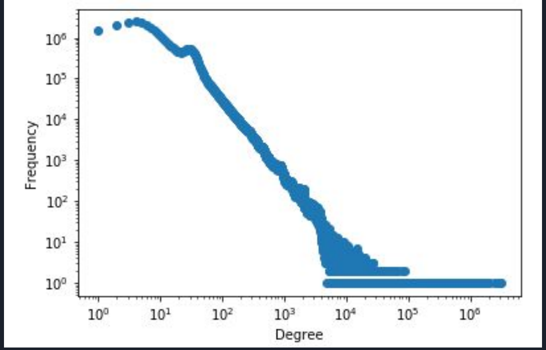
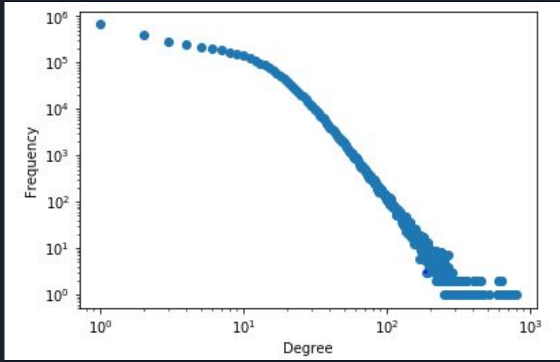
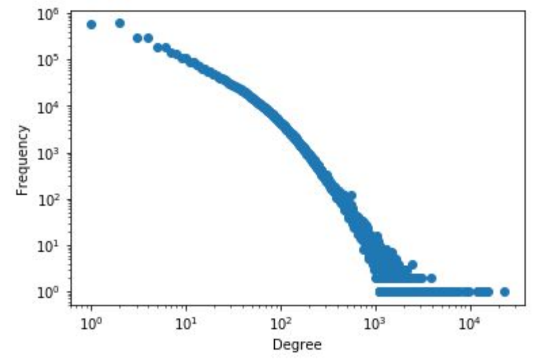
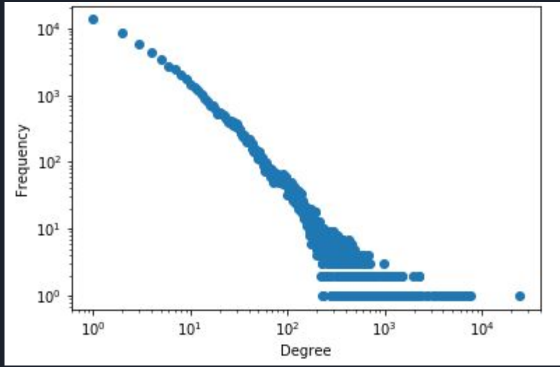


Smooth Kronecker looks like real graphs









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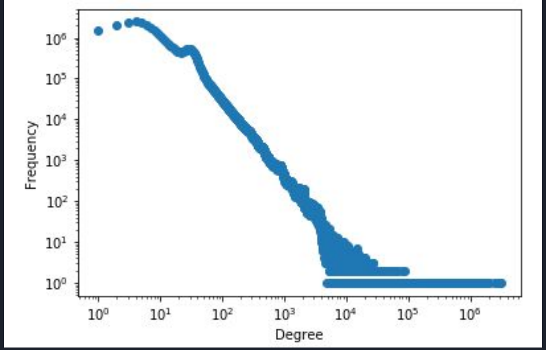
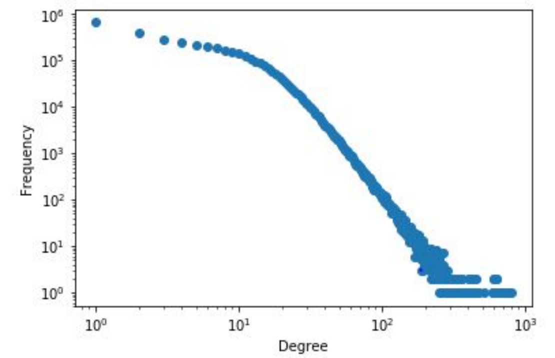


Twitter-2010



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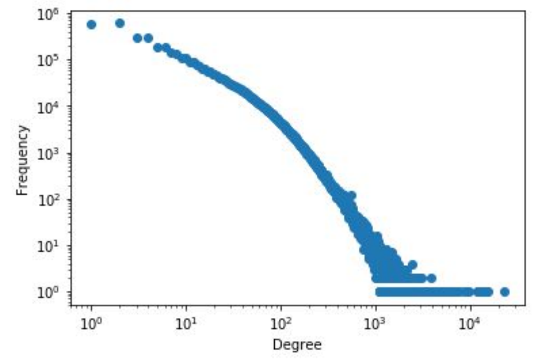

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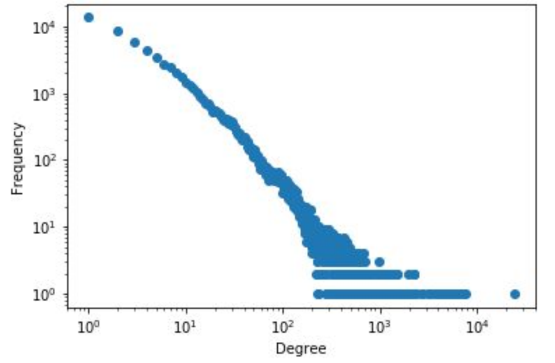
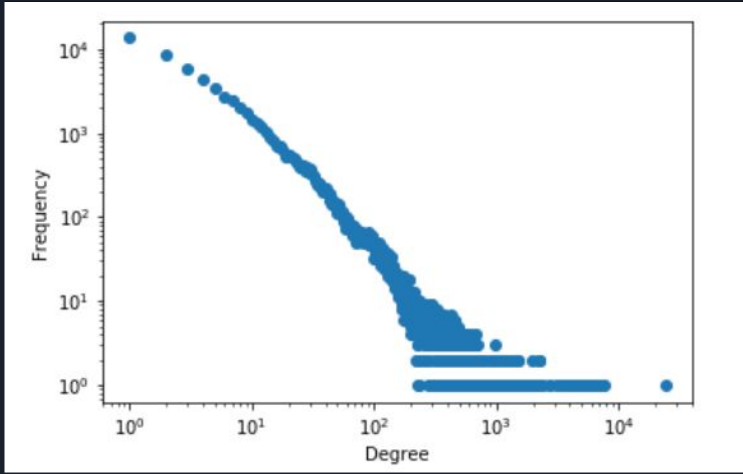


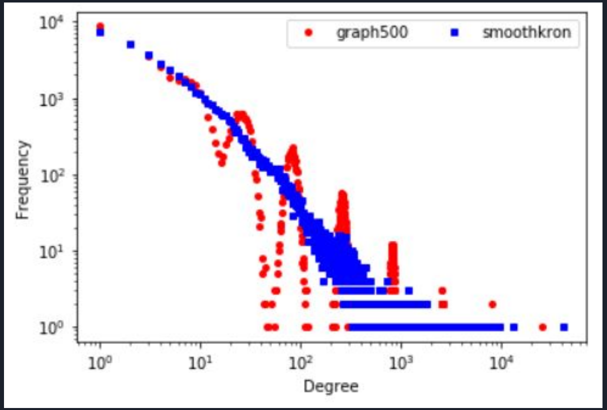
Smooth-Kron

Large Variety of Scales

- With Smooth Kronecker, we are no longer restricted to powers of 2. Current implementation supports number of vertices of the form $2^x * 3^y$.
- To achieve smoothening of the degree distribution, only 1 resampled seed is required.
- Smooth Kronecker provides finer control over the number of vertices for potential scalability experiments
- With other resamplings, other scales can also be supported.



Smooth Kron to rule them all



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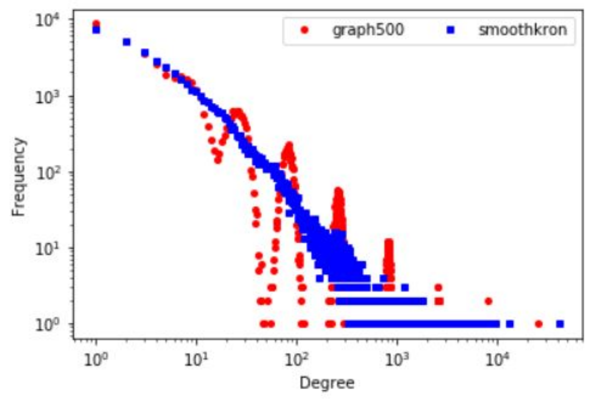

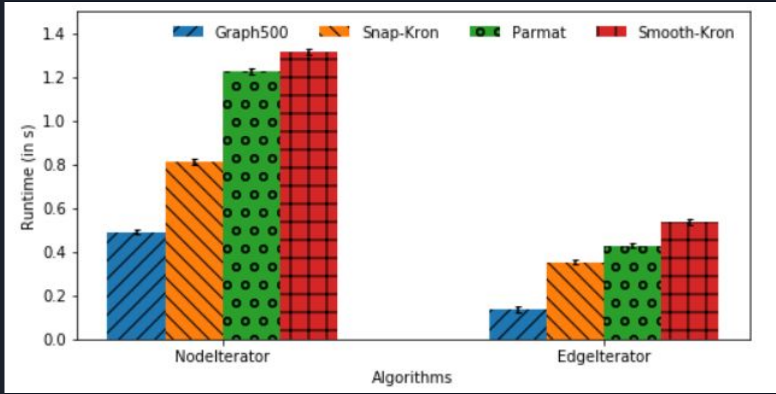

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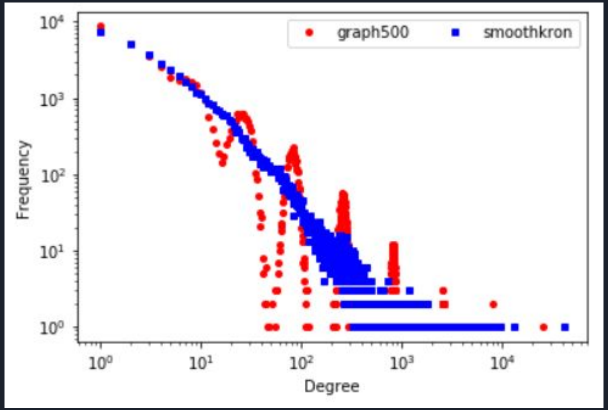
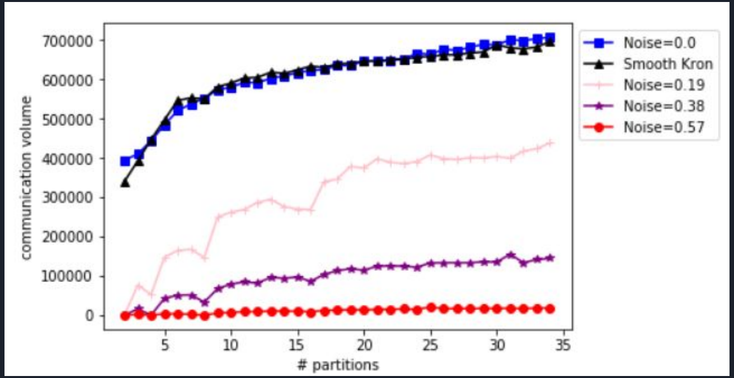
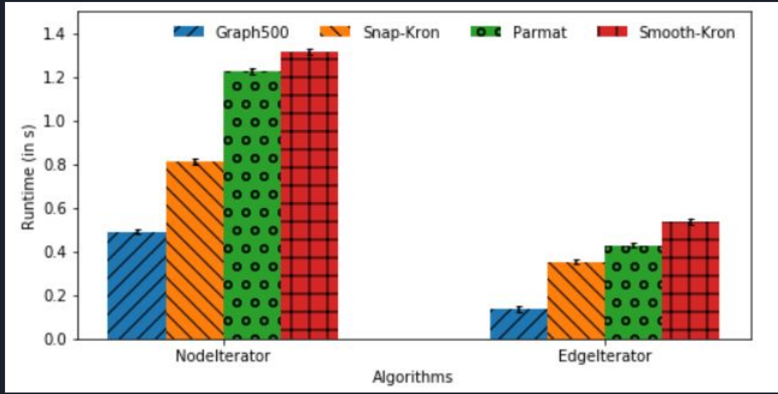
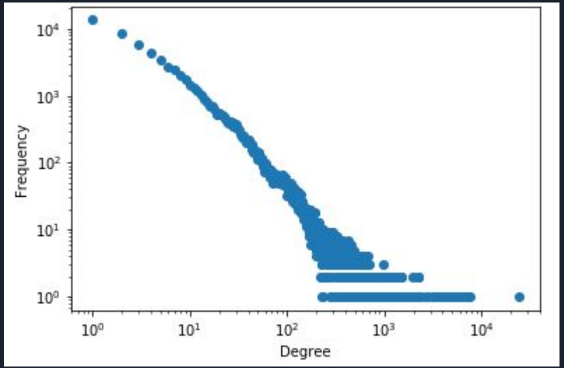






Conclusion

- We strongly urge the community to use the **Smooth Kronecker Generator** for generating synthetic graphs.
- **Smooth Kronecker Generator** is open source and available at https://github.com/dmargo/smooth_kron_gen



"Decrease Noise Pollution, Use Smooth Kronecker!"